Essential Plane Geometry Formulas

A concise school reference guide

Triangles, quadrilaterals, circles, polygons and coordinate geometry

1. Angles

Supplementary adjacent angles:

$$\alpha + \beta = 180^\circ.$$

Vertically opposite angles are equal.
When two parallel lines are crossed by a transversal:

- corresponding angles are equal;
- alternate interior angles are equal;
- same-side interior angles add up to 180°.

2. Triangle

The sum of the interior angles:

$$\alpha + \beta + \gamma = 180^\circ.$$

Triangle inequalities:

$$a + b > c,$$

$$b + c > a,$$

$$c + a > b.$$

Perimeter:

$$P = a + b + c.$$

Semiperimeter:

$$p = \frac{a + b + c}{2}.$$

3. Area of a triangle

Using a base and its corresponding height:

$$S = \frac{1}{2} a h_a.$$

Using two sides and the included angle:

$$S = \frac{1}{2} ab \sin \gamma.$$

Heron's formula:

$$S = \sqrt{p(p-a)(p-b)(p-c)}.$$

Using the inradius:

$$S = pr.$$

Using the circumradius:

$$S = \frac{abc}{4R}.$$

Here r is the radius of the inscribed circle, and R is the radius of the circumscribed circle.

4. Right triangle

The **hypotenuse** is the side opposite the right angle.
The **opposite leg** is the leg opposite the angle α.
The **adjacent leg** is the leg next to the angle α.
If c is the hypotenuse, the Pythagorean theorem gives:

$$a^2 + b^2 = c^2.$$

Sine of an angle:

$$\sin\alpha = \frac{\text{opposite leg}}{\text{hypotenuse}}.$$

Cosine of an angle:

$$\cos\alpha = \frac{\text{adjacent leg}}{\text{hypotenuse}}.$$

Tangent of an angle:

$$\tan\alpha = \frac{\text{opposite leg}}{\text{adjacent leg}}.$$

If the opposite leg is denoted by a, the adjacent leg by b, and the hypotenuse by c, then:

$$\sin\alpha = \frac{a}{c}, \qquad \cos\alpha = \frac{b}{c}, \qquad \tan\alpha = \frac{a}{b}.$$

The altitude h drawn to the hypotenuse is:

$$h = \frac{ab}{c}.$$

If m and n are the projections of the legs a and b onto the hypotenuse, then:

$$a^2 = cm,$$

$$b^2 = cn,$$

$$h^2 = mn.$$

5. Laws of sines and cosines

Law of sines:

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma} = 2R.$$

Law of cosines:

$$c^2 = a^2 + b^2 - 2ab\cos\gamma.$$

The other forms are obtained by cyclically permuting the sides and their opposite angles.

6. Median and angle bisector

The median to side a:

$$m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}.$$

An angle bisector divides the opposite side in the ratio of the adjacent sides:

$$\frac{BD}{DC} = \frac{AB}{AC}.$$

Length of the angle bisector:

$$l_a = \frac{\sqrt{bc\big((b+c)^2 - a^2\big)}}{b+c}.$$

7. Similar triangles

If the scale factor is k, then:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = k.$$

Ratio of the perimeters:

$$\frac{P_1}{P_2} = k.$$

Ratio of the areas:

$$\frac{S_1}{S_2} = k^2.$$

8. Parallelogram

Perimeter:

$$P = 2(a + b).$$

Area using a side and its corresponding height:

$$S = ah_a.$$

Area using two sides and the included angle:

$$S = ab \sin \alpha.$$

For the diagonals:

$$d_1^2 + d_2^2 = 2(a^2 + b^2).$$

The diagonals of a parallelogram bisect each other.

9. Rectangle and square

Rectangle:

$$S = ab.$$

$$P = 2(a + b).$$

$$d = \sqrt{a^2 + b^2}.$$

Square:

$$S = a^2.$$

$$P = 4a.$$

$$d = a\sqrt{2}.$$

10. Rhombus

Perimeter:

$$P = 4a.$$

Area using a side and its height:

$$S = ah.$$

Area using a side and an angle:

$$S = a^2 \sin \alpha.$$

Area using the diagonals:

$$S = \frac{d_1 d_2}{2}.$$

The diagonals of a rhombus are perpendicular and bisect its angles.

11. Trapezoid

Midsegment:

$$m = \frac{a + b}{2}.$$

Area:

$$S = \frac{a + b}{2} h.$$

Equivalently:

$$S = mh.$$

In an isosceles trapezoid, the diagonals are equal, and the angles adjacent to each base are equal.

12. Polygons

The sum of the interior angles of an n-gon:

$$\Sigma = (n-2)\cdot 180^\circ.$$

Each interior angle of a regular n-gon:

$$\alpha = \frac{(n-2)180^\circ}{n}.$$

Number of diagonals:

$$N = \frac{n(n-3)}{2}.$$

Area of a regular polygon:

$$S = \frac{1}{2}Pr,$$

where r is the radius of the inscribed circle, also called the apothem.

13. Circle and disk

Circumference:

$$L = 2\pi R = \pi d.$$

Area of a disk:

$$S = \pi R^2.$$

Arc length when the angle is measured in degrees:

$$l = \frac{\alpha}{360^\circ}\cdot 2\pi R.$$

When the angle is measured in radians:

$$l = R\alpha.$$

Area of a sector when the angle is measured in degrees:

$$S_{\text{sector}} = \frac{\alpha}{360^\circ}\pi R^2.$$

When the angle is measured in radians:

$$S_{\text{sector}} = \frac{1}{2}R^2\alpha.$$

14. Angles in a circle

A central angle is equal to the degree measure of the arc that it subtends.
An inscribed angle is equal to half the degree measure of the arc that it subtends:

$$\angle ACB = \frac{1}{2}\overset{\frown}{AB}.$$

The angle between a tangent and a chord is equal to the inscribed angle subtending the same arc.

15. Chords, secants and tangents

For two intersecting chords:

$$AM\cdot MB = CM\cdot MD.$$

For two secants drawn from the same external point:

$$PA\cdot PB = PC\cdot PD.$$

For a tangent and a secant drawn from the same point:

$$PT^2 = PA\cdot PB.$$

A radius drawn to the point of tangency is perpendicular to the tangent.

16. Coordinate geometry

Distance between two points:

$$AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}.$$

Coordinates of the midpoint:

$$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right).$$

Equation of a straight line:

$$y = kx + b.$$

Equation of a circle with centre (a, b):

$$(x-a)^2+(y-b)^2 = R^2.$$

How to choose an area formula

If a base and its corresponding height are known:

$$S = \frac{1}{2}ah.$$

If two sides and their included angle are known:

$$S = \frac{1}{2}ab\sin\gamma.$$

If all three sides are known, use Heron's formula.
If the inradius is known:

$$S = pr.$$

If the circumradius is known:

$$S = \frac{abc}{4R}.$$

This reference sheet is intended for quick review. Before using a formula, check the conditions under which it is valid and the meaning of each symbol.

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