Essential Algebra Formulas

A concise school reference guide

Powers, roots, identities, equations, functions, sequences and logarithms

1. Numbers, absolute value, proportions and percentages

Absolute value:

$$|a| = \begin{cases} a, & a \geq 0, \\ -a, & a < 0. \end{cases}$$

Properties of absolute value:

$$|ab| = |a||b|.$$

$$\left|\frac{a}{b}\right| = \frac{|a|}{|b|}, \qquad b \neq 0.$$

The quantity $|a - b|$ is the distance between the numbers a and b.
Proportion:

$$\frac{a}{b} = \frac{c}{d} \quad \Longleftrightarrow \quad ad = bc,$$

$$b \neq 0, \qquad d \neq 0.$$

Percentages:

$$p\% = \frac{p}{100}.$$

$$\text{part} = \text{whole} \cdot \frac{p}{100}.$$

$$\text{whole} = \frac{\text{part}}{p/100}, \qquad p \neq 0.$$

2. Powers

For values for which all expressions are defined:

$$a^m a^n = a^{m+n}.$$

$$\frac{a^m}{a^n} = a^{m-n}, \qquad a \neq 0.$$

$$(a^m)^n = a^{mn}.$$

$$(ab)^n = a^n b^n.$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, \qquad b \neq 0.$$

Zero exponent:

$$a^0 = 1, \qquad a \neq 0.$$

Negative exponent:

$$a^{-n} = \frac{1}{a^n}, \qquad a \neq 0.$$

Rational exponent:

$$a^{m/n} = \sqrt[n]{a^m}, \qquad a \geq 0, \quad n \in \mathbb{N}.$$

3. Roots

$$(\sqrt{a})^2 = a, \qquad a \geq 0.$$

$$\sqrt{a^2} = |a|.$$

$$\sqrt{ab} = \sqrt{a}\sqrt{b}, \qquad a \geq 0, \quad b \geq 0.$$

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}, \qquad a \geq 0, \quad b > 0.$$

For an even-indexed root, the radicand must be non-negative.

4. Algebraic fractions

Provided that all denominators are non-zero:

$$\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd}.$$

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}.$$

$$\frac{a}{b} : \frac{c}{d} = \frac{ad}{bc}.$$

For division, it is also necessary that:

$$c \neq 0.$$

Only factors may be cancelled:

$$\frac{ab + ac}{a} = \frac{a(b + c)}{a} = b + c, \qquad a \neq 0.$$

5. Special product identities

Square of a sum:

$$(a + b)^2 = a^2 + 2ab + b^2.$$

Square of a difference:

$$(a - b)^2 = a^2 - 2ab + b^2.$$

Difference of squares:

$$(a - b)(a + b) = a^2 - b^2.$$

Cube of a sum:

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3.$$

Cube of a difference:

$$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3.$$

Sum of cubes:

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2).$$

Difference of cubes:

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2).$$

6. Linear equation

General form:

$$ax + b = 0.$$

If $a \neq 0$, then:

$$x = -\frac{b}{a}.$$

If

$$a = 0, \qquad b = 0,$$

the equation has infinitely many solutions.
If

$$a = 0, \qquad b \neq 0,$$

the equation has no solutions.

7. System of two linear equations

$$\begin{cases} a_1x + b_1y = c_1, \\ a_2x + b_2y = c_2. \end{cases}$$

The determinant of the system is:

$$\Delta = a_1b_2 - a_2b_1.$$

If $\Delta \neq 0$, the system has a unique solution:

$$x = \frac{c_1b_2 - c_2b_1}{\Delta}.$$

$$y = \frac{a_1c_2 - a_2c_1}{\Delta}.$$

8. Quadratic equation

General form:

$$ax^2 + bx + c = 0, \qquad a \neq 0.$$

Discriminant:

$$D = b^2 - 4ac.$$

If $D \geq 0$, the roots are calculated using:

$$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a}.$$

If $D > 0$, the equation has two distinct real roots.
If $D = 0$, the equation has one real root:

$$x = -\frac{b}{2a}.$$

If $D < 0$, the equation has no real roots.

9. Vieta's formulas

For the quadratic equation

$$ax^2 + bx + c = 0, \qquad a \neq 0,$$

the roots satisfy:

$$x_1 + x_2 = -\frac{b}{a}.$$

$$x_1 x_2 = \frac{c}{a}.$$

If the equation has roots x_1 and x_2, then:

$$ax^2 + bx + c = a(x - x_1)(x - x_2).$$

10. Quadratic function

General form:

$$y = ax^2 + bx + c, \qquad a \neq 0.$$

The x-coordinate of the vertex is:

$$x_0 = -\frac{b}{2a}.$$

The y-coordinate of the vertex is:

$$y_0 = f(x_0) = -\frac{D}{4a}.$$

Axis of symmetry:

$$x = -\frac{b}{2a}.$$

If $a > 0$, the parabola opens upwards.
If $a < 0$, the parabola opens downwards.

11. Inequalities

When both sides of an inequality are multiplied or divided by a negative number, the inequality sign is reversed:

$$a < b, \qquad c < 0 \quad \Longrightarrow \quad ac > bc.$$

A quadratic inequality can be solved conveniently using the interval method after factoring the expression:

$$a(x - x_1)(x - x_2) \gtrless 0.$$

The zeros of the expression form the boundaries of the intervals.

12. Arithmetic sequence

The n-th term:

$$a_n = a_1 + (n-1)d.$$

The sum of the first n terms:

$$S_n = \frac{n(a_1 + a_n)}{2}.$$

Equivalently:

$$S_n = \frac{n}{2}\big(2a_1 + (n-1)d\big).$$

Here d is the common difference of the arithmetic sequence.

13. Geometric sequence

The n-th term:

$$b_n = b_1 q^{n-1}.$$

The sum of the first n terms:

$$S_n = b_1 \frac{q^n - 1}{q - 1}, \qquad q \neq 1.$$

If

$$|q| < 1,$$

the sum of the infinite geometric sequence is:

$$S_\infty = \frac{b_1}{1 - q}.$$

Here q is the common ratio of the geometric sequence.

14. Logarithms

Definition of a logarithm:

$$\log_a b = c \quad \Longleftrightarrow \quad a^c = b,$$

where

$$a > 0, \qquad a \neq 1, \qquad b > 0.$$

Logarithm of a product:

$$\log_a(xy) = \log_a x + \log_a y.$$

Logarithm of a quotient:

$$\log_a \frac{x}{y} = \log_a x - \log_a y.$$

Logarithm of a power:

$$\log_a x^k = k \log_a x.$$

Change of base:

$$\log_a b = \frac{\log_c b}{\log_c a},$$

where

$$c > 0, \qquad c \neq 1.$$

Basic values:

$$\log_a 1 = 0.$$

$$\log_a a = 1.$$

15. Basic functions

Linear function:

$$y = kx + b.$$

Inverse proportionality:

$$y = \frac{k}{x}, \qquad x \neq 0.$$

Absolute value function:

$$y = |x|.$$

Square root function:

$$y = \sqrt{x}, \qquad x \geq 0.$$

Exponential function:

$$y = a^x, \qquad a > 0, \quad a \neq 1.$$

Logarithmic function:

$$y = \log_a x, \qquad a > 0, \quad a \neq 1, \quad x > 0.$$

Before giving your answer

Check the domain of every expression, the signs, the expansion of brackets, and substitute the roots back into the original equation.

This reference sheet is intended for quick review. Before using a formula, check the conditions under which it is valid and the domain of all expressions.